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ROBUST OPTIMIZATION IN PORTFOLIO SELECTION BY m-MAD MODEL APPROACH

Abstract: *The portfolio selection problem is one of the main investment management problems. In the portfolio selection problem, robustness is sought against uncertainty or variability in the value of the parameters of the problem. In this paper, an extended mean absolute deviation model named the m-MAD model is applied to construct a new robust portfolio selection model that is solvable to real-world problems. The m-MAD model is a linear programming model and allows us to measure risk using downside deviations with the ability to penalize larger downside deviations. It also has a better performance of risk-averse priorities. The results of the performance analysis of the model show that the solutions of the m-MAD model are compatible with respect to second-degree stochastic dominance.*

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JEL Classification: C61, G11

1. Introduction

Portfolio optimization is the process of analyzing a portfolio and managing the assets within it. Markowitz (1952) has presented the Modern Portfolio Theory (MPT) and tried to maximize the return and control risk through the minimization of the variance of the portfolio return as a risk measure. In spite of some advantages, the Markowitz model has two difficulties: (a) quadratic programming problems are more difficult to solve, and (b), for practical markets, the size of the covariance matrix for solving the model is very large and difficult to estimate. To

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overcome these difficulties, many researchers have tried to present linear programming for the portfolio selection problem. Konno and Yamazaki (1991) suggested the mean absolute deviation (MAD) for risk measure as a linear model. Absolute deviation in the MAD model is sensitive against any upward or downward movement of the mean. An investor who uses the MAD model is assumed to have a constant trade-off: a unit deviation from the mean rate of return. This assumption does not allow for the distinction of risk associated with larger losses. It can be argued that the variability of the rate of return above the mean should not be penalized. Since the investors are concerned with the underperformance of a portfolio rather than the over performance of a portfolio, Markowitz (1959) proposed downside risk measures. Subsequently, Michalowski and Ogryczak (2001) suggested the m-MAD model. The m-MAD model is a true downside risk measure that can distinguish larger losses. Researchers such as (Kellerer et al 2000, Mansini et al 2003, Chiodi et al 2003, Papahristodoulou and Dotzauer 2004, and Rockafellar and Uryasev 2000) extended the models by presenting similar ideas on the risk measure for a linear programming formulation. The parameters on the mentioned models are defined with their nominal value and it is assumed that all parameters are constant. However in the real world, we deal with the problems where robustness is sought against uncertainty or variability in the value of parameters of the problem.

In the recent years, a body of the literature is developing under the name of robust optimization to consider uncertainty in the value of parameters of the model. Soyster (1973) proposed a linear optimization model to construct a solution that is feasible for all data that belong in a convex set. The solutions of the Soyster model are too conservative in the sense and it causes to give up too much of optimality for the nominal problem in order to ensure robustness. The second step forward for developing a theory for robust optimization was taken independently by Ben-Tal and Nemirovski (2000) and El-Ghaoui et al (1998). They use ellipsoidal uncertainty set. This model can adjust the conservatism. However, this model is not linear which can be problematic in the real world problems. Another development on robust optimization has been done by Bertsimas and Sim (2004). This model is linear, applicable and extendable to discrete optimization and can flexibly adjust the level of conservatism of the robust solutions in terms of probabilistic bounds of constraint violations. In this paper, we use Bertsimas and Sim methodology for development of our model.

There are some practical models of robust optimization in finance. El-Ghaoui et al (2003) proposed a robust portfolio model under an uncertainty of covariance matrix witch is developed by semi-definite programming (SDP) and considers worst case value-at-risk. Tutuncu and Koenig (2004) developed a robust portfolio optimization problem formulated in a quadratic program (QP). Kawas and Theile (2008) developed a log robust portfolio model to consider the heavy tailed property of stock prices. Moon and Yao (2011) developed a robust mean absolute deviation model for portfolio optimization. Quaranta and Zaffaroni (2008) developed a robust optimization of conditional value at risk. Chen and Tan (2009) developed

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robust portfolio selection based on asymmetric measures of variability of stock returns. In this paper we develop a robust model for the m-MAD model. The m-MAD model is a linear and downside risk measure. The results of the m-MAD model are second order stochastic dominance (SSD).

The rest of the paper is organized as follows. In Section 2, we explain the MAD and m-MAD models. We propose robust optimization of m-MAD model in Section 3. The computation results of empirical study based on historical data are discussed in Section 4. Finally, the conclusion comes in Section 5.

2. m-MAD model

Let $J = \{1,2,...,n\}$ denotes a set of securities considered for investment. The rate of return for each security $j \in J$ is represented by a random variable R_j with a given mean $\mu_j = E(R_j)$.

Further, let $X = (x_1, x_2, ..., x_n)$ denotes a vector of securities weights (decision variables) defining a portfolio. The weights must satisfy a set of constraints that form a feasible set *Q*. The weights must sum to one and there is not short selling:

$$\left\{ X = (x_1, x_2, ..., x_n)^T : \sum_{j=1}^{n} x_j = 1, x_j \geq 0 \quad ; j = 1,...,n \right\} \tag{1}$$

Each portfolio X defines a corresponding random variable $R_X = \sum_{j=1}^{n} R_j x_j$ that represents a portfolio rate of return. The mean rate of return for portfolio *X* is given as:

$$\mu_{(X)} = E(R_X) = \sum_{j=1}^{n} \mu_j x_j$$

2.1. MAD model

Konno and Yamazaki (1991) tried to represent a linear risk measure. They define the mean absolute deviation from a mean as follows.

$$\delta(x) = E\left\{\left|R_x - \mu_{(x)}\right|\right\} = \int_{-\infty}^{+\infty} \left|\mu_{(x)} - \xi\right| P_x(d\xi) \tag{2}$$

Where P_X denotes a probability measure induced by the random variable R_X.
Many authors (Konno, 1999; Fienstein and Thapa, 1993; Zenios and Kang, 1993) have pointed out that the MAD model opens up opportunities for more specific modeling of the downside risk, because absolute deviation may be considered as a measure of downside risk. Namely, the mean absolute deviation $\delta_{(X)}$ equals twice the (downsides) absolute semi deviation.

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$$\bar{\delta}(x) = E(max\{\mu(x) - R_x, 0\}) \tag{3}$$

According to (Konno, 1999), the following parametric optimization problem is called the MAD model;

$$max\{\mu(x) - \lambda\bar{\delta}(x) : X \in Q\} \tag{4}$$

The proposed extension to the MAD model allows one to differentiate between downside and upside risk, and to penalize larger downside deviations. It thus provides for better modeling of risk adverse preferences. Note that such an extension is in some ways equivalent to replacing $\bar{\delta}_{(X)}$ with $\bar{\delta}_u(x)$ defined as:

$$\bar{\delta}_u(x) = E(u(max\{\mu(x) - R_x, 0\})) \tag{5}$$

Where, u is some convex penalty function.

It is assumed that r_{jt} is the realization of the random variables R_j during the period t (where *t=1,…,T*) that is available from historical data. It is also assumed that the expected value of R_j can be approximated by:

$$\mu_j = \frac{1}{T}\sum_{t=1}^{T} r_{jt}$$

Therefore, model (4) for a discrete set of realizations r_{jt} can be rewritten as the following LP model (Fienstein and Thapa, 1993):

$$max\sum_{j=1}^{n}\mu_j x_j - \tfrac{\lambda}{T}\sum_{t=1}^{T} d_t \tag{6}$$

Subject to

$$X \in Q \tag{7}$$

$$d_t \geq \sum_{j=1}^{n}(\mu_j - r_{jt})x_j \qquad ; t = 1,...,T \tag{8}$$

$$d_t \geq 0 \qquad ; t = 1,...,T \tag{9}$$

If the rate of return is normally distributed multivariate, then the MAD model is equivalent to the Markowitz model (Konno, 1999).

Recently, the MAD model was further validated by Ogryczak and Ruszczynski (1999). They demonstrated that if the trade-off coefficient λ is bounded by 1, then the MAD model is partially consistent with second degree stochastic dominance (Whitmore and Findlay, 1978).

2.2. Extended MAD Model

The extended MAD model is to differentiate between the various levels of downside deviations, and to penalize the larger ones. Konno (1999) has already proposed such an extension of the MAD model for portfolio optimization. He

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considered additional mean deviations from some target rate of return predefined as being proportional to the means rate of return:

$$\bar{\delta}_k(x) = E(max\{k\mu(x) - R_x, 0\}) \qquad for\, 0 \le k \le 1 \tag{10}$$

If $k=1$ then $\bar{\delta}_1(x) = \bar{\delta}(x)$ and this model is like absolute semi deviation used in the original MAD model. One may attempt to augment the downside risk measure by penalizing additional deviations for several $k<1$. In terms of penalty function (5), this approach is equivalent to introducing a convex piecewise linear function with break points proportional to the mean of R_x. Konno (1999) developed MAD model with additional downside deviation as follow:

$$max\{\mu(x) - \lambda\bar{\delta}(x) - \lambda_k\bar{\delta}_k(x) : X \in Q\} \tag{11}$$

Where, $\lambda > 0$ is the basic trade-off parameter and $\lambda_k > 0$ is an additional parameter. We refer to this model as k-MAD.

Assuming that λ has value T_1. Mean surplus deviation $E\{max\{\mu(x) - \bar{\delta}(x) - R_x, 0\}\}$ needs to be penalized by a value, let's say T_2, of a trade-off between the surplus deviation and a mean deviation, which leads to maximization of the following equation:

$$\mu(x) - T_1(\bar{\delta}(x) + T_2E(max\{\mu(x) - \bar{\delta}(x) - R_x, 0\})) \tag{12}$$

One might wish to penalize second level surplus deviation exceeding that mean.

$$max\left\{\mu(x) - \sum_{i=1}^{m}(\prod_{k=1}^{i} T_k)\bar{\delta}_i(x) : X \in Q\right\} \tag{13}$$

Where, $T_1 > 0, \ldots, T_m > 0$ are the assumed known trade-off coefficients. Trade-off coefficients are measured as follow:

$$\lambda_i = \prod_{k=1}^{i} T_k \qquad ; i = 1, \ldots, m \tag{14}$$

The model formulated as follow:

$$max\left\{\mu(x) - \sum_{i=1}^{m}\lambda_i\bar{\delta}_i(x) : X \in Q\right\} \tag{15}$$

We will refer to model (15) as the recursive m-level MAD model (Michalowski and Ogryczak, 2001).

If trade-off coefficients are positive and not greater than one and satisfying:

$$1 \ge \lambda_1 \ge \ldots \ge \lambda_m > 0 \tag{16}$$

Then the results of the m-MAD model will be SSD (Michalowski and Ogryczak, 2001). In addition, the m-MAD model with m=2 is formulated as an LP problem.

$$max\sum_{J=1}^{n}\mu_j x_j - \frac{\lambda_1}{T}\sum_{t=1}^{T} d_{t1} - \frac{\lambda_2}{T}\sum_{t=1}^{T} d_{t2} \tag{17}$$

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Subject to

$$X \in Q \tag{18}$$

$$d_{t1} \geq \sum_{j=1}^{n}(\mu_j - r_{jt})x_j \quad ;t = 1,...,T \tag{19}$$

$$d_{t2} \geq \sum_{j=1}^{n}(\mu_j - r_{jt})x_j - \frac{1}{T}\sum_{L=1}^{T} d_{L1} \quad ;t = 1,...,T \tag{20}$$

$$d_{t1} \geq 0, d_{t2} \geq 0 \quad ;t = 1,...,T \tag{21}$$

A general m-MAD model can be formulated as a LP model.

$$max\, w_0 + \sum_{i=1}^{m} \lambda_i w_i \tag{22}$$

Subject to

$$X \in Q \tag{23}$$

$$w_0 - \sum_{j=1}^{n} \mu_j x_j = 0 \tag{24}$$

$$Tw_i + \sum_{t=1}^{T} d_{ti} = 0 \quad ;\ i = 1,...,m \tag{25}$$

$$d_{ti} - \sum_{k=0}^{i=1} w_k + \sum_{j=1}^{n} r_{jt} x_j \geq 0 \quad ;t = 1,...,T, \quad i = 1,...,m \tag{26}$$

$$d_{ti} \geq 0 \quad ;t = 1,...,T, \quad i = 1,...,m \tag{27}$$

In the above formulation $\mu(x)$ and $\overline{\delta}_i(x)$ are explicitly represented using additional variables w_0 and $-w_i$.

3. Robust m-MAD model

This section develops a robust reformulation of the m-MAD model. There are three kinds of robust optimization based on uncertainty sets. Soyster (1973), Ben-Tal and Nemirovski (2000), and Bertsimas and Sim (2004) developed Robust Optimization. The result of Soyster model produces solutions that are too conservative. Ben-Tal and Nemirovski (2000) assumed that the uncertainty sets are ellipsoid. With this uncertainty set the robust counterparts are nonlinear; the last model of robust was developed by Bertsimas and Sim (2004). The robust counterparts of Bertsimas and Sim are linear.In m-MAD model, the expected return μ_j, of asset j is approximated by $\mu_j = \frac{1}{T}\sum_{t=1}^{T} r_{jt}$, which means that an actual

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return cannot be exactly obtained and has uncertainty. Let J the set of coefficients $\mu_j, j \in J$ that are subject to parameter uncertainty. $\tilde{\mu}_j, j \in J_0$ take values according to symmetric distribution with mean equal to the nominal value μ_j in the interval $[\mu_j - \hat{\mu}_j, \mu_j + \hat{\mu}_j]$. We introduce a parameter Γ_0, not necessary integer that takes values in the interval $[0,|j_0|]$. The role of the parameter Γ_0 is to adjust the robustness of the proposed model against the level of conservatism of the solution. For the chosen Γ_0, consider a subset S_0 satisfying the conditions $S_0 \subseteq J_0$ and $|S_0| = \lfloor \Gamma_0 \rfloor$.

For the given set S_0 and a coefficient $\hat{r}_v$, where, $v \notin S_0 \setminus J_0$ we like to allow a certain level of deviations in constraints. It is clear by the construction of robust formulation that if up to $\lfloor \Gamma_0 \rfloor$ of the J_0 coefficients μ_j change within their bounds, and up to one coefficient $\hat{\mu}_v$ changes by $(\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{\mu}_v$, and then the solution of problem will remain feasible and flexible. In other words, we stipulate that nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely effect on solution (Bertsimas and Sim, 2004).

There are two places in the m-MAD model that μ_j exists. At first we should change objective function to constraint. The form of constraints has to be like $ax \le b$ so the m-MAD model changes to:

$$max\ k \tag{28}$$

Subject to

$$X \in Q \tag{29}$$

$$-\sum_{j=1}^{n} \mu_j x_j - \sum_{i=1}^{m} \lambda_i w_i \le -k \tag{30}$$

$$Tw_i + \sum_{t=1}^{T} d_{ti} = 0 \qquad ; i = 1,\ldots,m \tag{31}$$

$$\sum_{j=1}^{n} \mu_j x_j + \sum_{k=1}^{i=1} w_k - \sum_{j=1}^{n} r_{jt} x_j \le d_{ti} \qquad ; t = 1,\ldots,T, \quad i = 1,\ldots,m \tag{32}$$

$$d_{ti} \ge 0 \qquad ; t = 1,\ldots,T, \quad i = 1,\ldots,m \tag{33}$$

We use the following formulation for (32). This formulation is exactly used for (30).

$$max\ k \tag{34}$$

Subject to

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$$X \in Q \tag{35}$$

$$-\sum_{j=1}^{n}\mu_j x_j - \sum_{i=1}^{m}\lambda_i w_i + \max_{\{s_0 \cup \{v\} | s_0 \subseteq j_0, |s_0| = \lfloor \Gamma_0 \rfloor, v \in j_0 \setminus s_0\}} \left\{ \sum_{j \in s_0} \hat{\mu}_j y_j + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{\mu}_v y_j \right\} \le -k \tag{36}$$

$$Tw_i + \sum_{t=1}^{T} d_{t_i} = 0 \qquad ; i = 1,\dots,m \tag{37}$$

$$\sum_{j=1}^{n}\mu_j x_j + \sum_{k=1}^{i=1} w_k - \sum_{j=1}^{n} r_{jt} x_j \max_{\{s_0 \cup \{v\} | s_0 \subseteq j_0, |s_0| = \lfloor \Gamma_0 \rfloor, v \in j_0 \setminus s_0\}} \left\{ \sum_{j \in s_0} \hat{\mu}_j y_j + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{\mu}_v y_j \right\} \le d_{ti} \qquad ; t = 1,\dots,T, \quad i = 1,\dots,m \tag{38}$$

$$d_{t_i} \ge 0 \qquad ; t = 1,\dots,T, \quad i = 1,\dots,m \tag{39}$$

Note that μ_j in (36) and (38) are same and we can use just one Γ_0 for both of them.

If Γ_0 is chosen as an integer:

$$B_0(x, \Gamma_0) = \max_{\{s_0 | s_0 \subseteq J_0, |s_0| = \Gamma_0\}} \left\{ \sum \hat{\mu}_j |x_j| \right\} \tag{40}$$

We need the follow proposition that reformulated (34) to (39) as a linear constraint. For given vector x^*:

$$B_0(x^*, \Gamma_0) = \max_{\{s_0 \cup \{v\} | s_0 \subseteq J_0, |s_0| = \lfloor \Gamma_0 \rfloor, v \in J_0 \setminus s_0\}} \left\{ \sum_{j \in s_0} \hat{\mu}_j |x_j^*| + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{\mu}_v |x_j^*| \right\} \tag{41}$$

The above formulation is equal to following linear optimization problem:

$$B_0(x^*, \Gamma_0) = \max \sum_{j \in J_0} \hat{\mu}_j |x_j^*| z_j \tag{42}$$

subject to:

$$\sum_{j \in J_0} Z_j \le \Gamma_0 \tag{43}$$

$$0 \le Z_j \le 1 \qquad \forall j \in J_0 \tag{44}$$

We consider dual formulation of above linear optimization problem as follow:

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$$min \sum_{j \in J_0} P_j + \Gamma_0 z_0 \tag{45}$$

Subject to:

$$Z_0 + P_j \geq \hat{\mu}_j \left| x_j^* \right| \qquad \forall j \in J_0 \tag{46}$$

$$P_j \geq 0 \qquad \forall j \in J_0 \tag{47}$$

$$Z_0 \geq 0 \tag{48}$$

By strong duality, since problems (42) to (44) are feasible and bounded for all $\Gamma_0 \in [0, |J_0|]$, the dual problems (45) to (48) are also feasible and bounded and their objective values coincide. In addition, $B_0(x^*, \Gamma_0)$ is equal to (36) and (38). Therefore we can reformulate the robust m-MAD model based on Bertsimas and Sim (2004) as follow:

$$max\ k \tag{49}$$

Subject to

$$X \in Q \tag{50}$$

$$-\sum_{j=1}^{n} \mu_j x_j - \sum_{i=1}^{m} \lambda_i w_i + z_0 \Gamma_0 + \sum_{j=1}^{n} P_j \leq -k \tag{51}$$

$$Tw_i + \sum_{t=1}^{T} d_{ti} = 0 \qquad ; i = 1, \dots, m \tag{52}$$

$$\sum_{j=1}^{n} \mu_j x_j + \sum_{k=1}^{i=1} w_k - \sum_{j=1}^{n} r_{jt} x_j + z_0 \Gamma_0 + \sum_{j=1}^{n} P_j \leq d_{ti} \qquad ; t = 1, \dots, T, \quad i = 1, \dots, m \tag{53}$$

$$z_0 + P_j \geq \hat{R}_j y_j \tag{54}$$

$$d_{ti} \geq 0 \qquad ; t = 1, \dots, T, \quad i = 1, \dots, m \tag{55}$$

$$d_{ti} \geq 0 \qquad ; t = 1, \dots, T; i = 1, \dots, m \tag{56}$$

$$P_j \geq 0 \qquad \forall j \in J_0, \qquad Z_0 \geq 0 \tag{57}$$

$$-y_j \leq x_j \leq y_j, \qquad y_j \geq 0 \qquad j = 1, \dots, N \tag{58}$$

4. Computational result

In this section, we show how robust optimization approach can be implemented to the m-MAD model. We show that the robust optimization of the m-MAD model can measure down side mean absolute deviation with uncertainty coefficients. We use real data from New York financial market. The data comes from New York stock exchange between April, 2012 and April 1, 2013 for 10 stocks.

The stocks that we use in this case study is as follow:

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Amazon, bank of America, bank of Montreal, Exxon Mobil, face book, FedEx, ford, general electric, general motors and yahoo
X1 to X10 respectively refer to above stocks.
Summary of data is in table 1:

Table1: Summary of Data from New York stock exchange

T	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	-0.081	-0.091	-0.001	-0.083	0	0.01	-0.063	-0.025	-0.034	0.047
2	0.072	0.111	0.035	0.088	0.05	0.029	-0.092	0.101	-0.111	0.005
3	0.021	-0.101	0.048	0.015	-0.301	-0.014	-0.036	-0.004	-0.0005	0.051
4	0.064	0.087	0.022	0.011	-0.168	-0.029	0.016	-0.001	0.083	-0.097
5	0.024	0.106	0.009	0.047	0.199	-0.032	0.055	0.104	0.065	0.062
6	-0.084	0.055	0.013	-0.002	-0.025	0.087	0.137	-0.072	0.12	0.032
7	0.082	0.058	0.015	-0.027	0.032	-0.025	0.026	0.003	0.014	0.126
8	-0.004	0.178	0.021	-0.018	-0.049	0.024	0.131	0.002	0.113	0.018
9	0.058	-0.025	0.043	0.039	0.163	0.106	0.006	0.061	-0.025	0.003
10	-0.004	-0.007	-0.015	0.001	-0.12	0.039	-0.026	0.05	-0.033	0.083
11	0.008	0.084	0.011	0.006	-0.061	-0.067	0.042	-0.004	0.024	0.152
12	-0.011	0.037	-0.026	-0.022	0.015	-0.058	-0.006	-0.076	0.051	-0.03
μ_j	0.012	0.041	0.006	0.055	0.002	0.005	0.015	0.011	0.022	0.038

We want to share investment between these 10 stocks. The penalty parameters are calculated as follow:

$\lambda_1 = \lambda \quad and \quad \lambda_2 = \lambda^2$

We consider $\lambda = 0.5$ and then we have $\lambda_1 = 0.5$ and $\lambda_2 = 0.25$.

We coded the robust m-MAD model by lingo software as a linear programming then we solved that with different robust price parameter. For uncertainty parameter we consider 20 percent volatility for each μ_j. We summarize the results of objective function in Table 2. In this table we show the value of the objective function and the price of robustness and the value of decision variables.

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Table 2: The value of the objective function for various price of robustness

Γ_i	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Objective Function
0	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02224
0.1	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02209
0.2	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02181
0.3	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02160
0.4	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02139
0.5	0.15	0.015	0.05	0.2	0	0.05	0.05	0.05	0.15	0.15	0.02117
0.6	0.1	0.1	0.075	0.2	0	0.075	0.075	0.075	0.1	0.2	0.2096
0.7	0.1	0.1	0.075	0.2	0	0.075	0.075	0.075	0.1	0.2	0.02082
0.8	0.1	0.1	0.075	0.2	0	0.075	0.075	0.075	0.1	0.2	0.02068
0.9	0.083	0.1	0.083	0.2	0	0.083	0.083	0.083	0.083	0.2	0.02055
1	0.083	0.1	0.083	0.2	0	0.083	0.083	0.083	0.083	0.2	0.02043
2	0.075	0.0875	0.0875	0.2	0	0.0875	0.0875	0.0875	0.0875	0.2	0.01959
3	0.071	0.0890	0.0890	0.2	0	0.0890	0.0890	0.0890	0.0890	0.1936	0.01889
4	0.0598	0.0950	0.0950	0.2	0	0.0950	0.0950	0.0950	0.0950	0.1696	0.01853
5	0	0.0938	0.0938	0.2	0.0623	0.0938	0.0938	0.0938	0.0938	0.01747	0.01845
6	0	0.0938	0.0938	0.2	0.0623	0.0938	0.0938	0.0938	0.0938	0.01747	0.01845
10	0	0.0938	0.0938	0.2	0.0623	0.0938	0.0938	0.0938	0.0938	0.01747	0.01845

In this table we show different solution based on different price of robustness. As we show in table by increase of C the objective function reduced. The first row is about the situation that $\Gamma_i=0$. It means there isn't any uncertainty in parameters. When $\Gamma_i=0$ the result is equal to the situation that we don't use robust approach. This row show different between the use of robust approach and original m-MAD. X4 has the best rate of return in this set of stocks and x5 refer to face book rate of return that has the worst rate of return in this set. As shown in table the model try to maximize x4 because it has the best effect on portfolio and all situation of Γ_i, x4 has the most share in portfolio and x5 has the less share in portfolio. From $\Gamma_i=0$ to $\Gamma_i=1$ there is a little different between solution because we use small Γ_i .investors can use this strategy if they anticipate small volatility in market.

From row $\Gamma_i=1$ to $\Gamma_i=10$ the uncertainty of parameters goes up. And the solutions have big change. But from $\Gamma_i=5$ to $\Gamma_i=10$ the solution don t change any more. There is a mathematical explain for this phenomena. Robust optimization consider the optimum solution in worst case of uncertainty and from $\Gamma_i=5$ to $\Gamma_i=10$ the best solution that remain feasible are achieved.

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Based on information in the table, by increase of level of robustness the level of conservatism increases and objective function is reduced.

As we show the robust method of m-MAD model that we represent, consider uncertainty coefficient in the good way. Experimental results show that by increasing the level of robustness how the model is reacting. When, the price of robustness increases, the conservatism of solution has increase.

5. Conclusions

This paper developed a new robust model in portfolio optimization by using the m-MAD approach. This model has advantages on computational complexity and provides robust solutions under parameter uncertainty .In addition, we shown conservatism of objective function against the level of robustness in this model. For future development, we offer use of robust optimization in goal programming for portfolio selection problem.

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